A REVISED SARGENT-WALLACE MODEL AND THE MANAGEMENT OF EXPECTATIONS

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ABSTRACT. The Sargent-Wallace model assumes that the public and policy authority share the same information set and policy can be expected. According to rational inattention, sticky information, and sticky expectations, even if policy authority released more transparent information and opened its policy, these two assumptions of the Sargent-Wallace model still would not work. Our paper adds a loss function and incorporates the management of expectations into the Sargent-Wallace model to fix its problems.

Keywords: Management of Expectations; Sticky Expectations; Rational Expectations; Rational Inattention; Sticky Information

1. Introduction. Sargent and Wallace (1975, 1976) proposed a new classical theory based upon the theory of rational expectations. It posited that monetary policy could not systematically manage the levels of output and employment in the economy.

The core idea of the Sargent-Wallace model is that the policy can be expected by the public. This idea is based on an assumption that the public and policy authority share the same information set. This assumption is too strong in many empirical cases. In the 1990s, many central banks released more transparent information to the public. This is helpful in supporting the assumption of the model.

According to the thought of Sims (1998), Mankiw and Reis (2002), and Carroll (2005), our paper argues that even if a central bank released more transparent information and opened its policy, the public would be unable to profit from the more transparent information since their decisions are not always based on all current available information. We think the management of expectations can help the public gets profit from the more transparent information.

Morris and Shin (2008) indicated: “According to the management of expectations, monetary policy is -- at its heart -- the problem of managing and coordinating expectations in the economy.” “Charles Goodhart has coined the term “expectationalists” to denote this school of thought that includes not only Michael Woodford, but other leading monetary economists such as Alan Blinder, Lars Svensson, and Ben Bernanke.”

Morris and Shin (2008) summarize basic thought of the management of expectations. According to this basic thought, we add a loss function into the Sargent-Wallace model.
Thus we can incorporate the management of expectations into this model. This revised Sargent-Wallace model can be used as a basic model of the management of expectations.

2. A Revised Sargent-Wallace Model with Optimal Control. To display how the management of expectations can be introduced into the Sargent-Wallace model (1975, 1976), we add a loss function on that familiar framework to get a revised Sargent-Wallace model.

\[ Q_t = aX_t - bP_t \]  
\[ S_t = Y + c(P_t - P^e_t) + U_t \]  
\[ P^e_t = E[P_t | I_{t-1}] \]  
\[ Q_t = S_t = Y_t \]  
\[ M.S.E. = E \sum_{t=0}^{\infty} \beta^t [(P_t - P^*)^2 + (Y_t - Y)^2] \]

Where, \( P_t \) is price level. \( X_t \) is policy variable. \( Q_t \) is gross demand. \( S_t \) is gross supply. \( Y \) is potential supply. \( P^e_t \) is the expectation of price level. \( U_t \) is white noise. E is a mathematical expectation operator. \( I_{t-1} \) is the information set. \( Y_t \) is the equilibrium gross demand and supply. \( P^* \) is a target of inflation. M.S.E. stands for the mean squared error of the loss function. \( \beta^t \) is the utility discount factor, and \( 0 < \beta < 1 \). Where a, b, and c are positive coefficients.

Based on the Sargent-Wallace model, let us suppose that the objective of policy is to minimize a loss function of the equation (5). Thus the Sargent-Wallace model becomes a control system. Policy authority controls \( X_t \) to try to make the mean squared error of the loss function to be as close to zero as possible.

Solving the equations, we get:

\[ Y_t - Y = \frac{ac(X_t - E[X_t | I_{t-1}]) + bu_t}{b+c+bu_t} \]  
\[ P_t - P^e_t = \frac{a(X_t - E[X_t | I_{t-1}]) - u_t}{b+c+bu_t} \]  
\[ P^e_t = \frac{aE[X_t | I_{t-1}]-Y}{b} \]

If policy can be expected, then it can be expressed by

\[ X_t = E[X_t | I_{t-1}] \]

If equation (9) is true, then we can get the following the results:

\[ Y_t - Y = \frac{bu_t}{b+c} \]  
\[ P_t - P^e_t = \frac{u_t}{b+c} \]

The original analysis of the Sargent-Wallace model focused on the debate over that monetary authority is powerless to combat the business cycle. In other words, policy is ineffective. According to their analysis, the key is whether policy can be expected. Of course, this is also a key problem of our revised model.

If we can control \( P^e_t \) of equation (8), then we can let \( p^* = p^e \). This means we have

\[ P^* = \frac{aE[X_t | I_{t-1}]-Y}{b} \]

This is another key problem that our revised Sargent-Wallace model focuses on. It is also a key problem that the management of expectations focuses on. Thus this is a key that we incorporate the management of expectations into the Sargent-Wallace model.

From equations (1)-(12) that as long as \( X_t = E[X_t | I_{t-1}] \) and \( p^* = p^e \) at all times, one possible optimal equilibrium is that the inflation gap (gap between real inflation and target
inflation) and the output gap are both zero at all times. And in such a case, this is obviously the equilibrium that minimizes the loss function (5). But it is possible for $X_t \neq E[X_t|I_{t-1}]$ and $p^* \neq p^c$. In such a case, the equilibrium that minimizes the loss function (5) is no longer a possibility.

How to let $X_t = E[X_t|I_{t-1}]$ and $p^* = p^c$? Different economics theory answers these two problems differently.

3. More Transparent Information. The assumptions of the Sargent-Wallace model are based on the three assumptions of the theory of rational expectations: (1) All agents use rational expectations. (2) All agents make efficient use of all available information. (3) All agents have instantaneous access to all the latest economic information. This set of assumptions has simplified structures of economic models, but it is too strong to explain the real world.

The theory of rational expectations thinks more transparent information would help fixing the problem that public and policy authority share the same information set. In fact, after 1990, many central banks have published more transparent information to the public.

4. Three New Theories. In recent 10 years, economists try to relax the three assumptions of rational expectations and form three new theories. They are rational inattention, sticky information and sticky expectations.

Sims (1998) presented a classic definition of rational inattention: “Because individuals have many things to think about and limited time, they can devote only limited intellectual resources to the tasks of data-gathering and analysis. We know from personal experience that many data that we could look up daily, and that are in principle relevant to our optimal economic decision-making, do not in fact influence our behavior, except when they change dramatically, or perhaps when we occasionally set aside some time to re-assess our portfolio.”

Mankiw and Reis (2002) indicated the core thought of sticky information: “The essence of the model is that information about macroeconomic conditions diffuses slowly through the population. This slow diffusion could arise because of either costs of acquiring information or costs to reoptimization. In either case, although prices are always changing, pricing decisions are not always based on current information.” “To formalize these ideas, we assume that each period a fraction of the population updates itself on the current state of the economy and computes optimal prices based on that information. The rest of the population continues to set prices based on old plans and outdated information.”

Dovern et al (2006) pointed out the basic idea of sticky expectations: “Microeconomic foundations for the sticky information paradigm were elaborated in Carroll’s (2003) work on the ‘epidemiological model of expectations.’ Carroll argues that US survey data on inflation expectations are consistent with a model in which, for each period, only a fraction of households adopts inflation forecasts of rational experts. The remaining households find it costly to update their information and continue using their past expectations rather than form fully rational predictions.”

The essence in rational inattention, sticky information and sticky expectations is that information diffuses slowly through the population, so expectation is sticky. According to
this thought, if decisions are not always based on current all available information, they are not be influenced quickly by more transparent information that policy authority published. Thus it is impossible that the public and policy authority share the same information set. And it is impossible that the policy can be expected by the public quickly. This is why we incorporate the management of expectations into the Sargent-Wallace model. We think this will help overcome these problems.

5. Managing Expectations. Based on our revised model, we can use some thought of the basic model of the management of expectations that Morris and Shin (2008) presented to analyze how policy authority uses these thought to make its policy and what benefit it can get.

In order to make the equation (9) true, i.e. policy can be expected, policy authority needs to communicate to the public, to focus on the pronouncements of its future actions. Morris and Shin (2008) pointed out: “One way of summarizing the modern expectationalist view of central banking is to say central banks have successfully taken over the role of sunspots. If economic actors can be persuaded that it is a central bank announcement, rather than the level of sunspot activity, that will coordinate expectations about interest rates and prices, and thus determine interest rates and inflation, then here is a free instrument for the central bank that offers a more predictable and smoother way of influencing outcomes than actually intervening in markets.”

In order to let $p^* = p^e$, policy authority need to improve its credibility and transparency. This makes public keep their expectations with the policy authority’s one. Morris and Shin (2008) indicated: “The “efficacy of central banking as sunspots” requires that central bank pronouncements acquire the same features as sunspots outlined above: they must be observed by all, it must be common knowledge that they are observed by all and there must be common knowledge of the exact meaning of the pronouncements.” These need policy authority to be both transparent and credible.

Equation (8) shows that the expectation of policy can influence expectations of price. This also means that changing policy can change expectations. Thus manipulating policy is still necessary for managing expectations.

According to our revised model, if policy authority uses management of expectations to make policy, the model will get the equilibrium solution that minimizes the loss function. This can eliminate unnecessary market uncertainties, volatility and increase efficiency of policy, and minimize the costs of policy. Thus policy authority does not want to mislead the public, and it does not want to use inflation policy to stimulate supply. Our result supports the thought of Woodford (2001): “It first considers the consequences of improved information about central-bank actions, and argues that the management of expectations will become even more important to effective monetary policy.”

6. Conclusion. We add a loss function and incorporate the management of expectations into the Sargent-Wallace model to fix its problems. We can use our revised Sargent-Wallace model as a framework to analyze how policy authority uses the management of expectations to make its policy and what benefit it can get.
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